Conceptualizing Division

California Common Core Standards

5.NBT.7 Multiply & Divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

5.NF.7b Interpret division of a whole number by a unit fraction, and compute such quotients.

6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions.

Objective

Students will be able to understand division using a semi-concrete model, and be able to apply that

understanding to more complex division problems, using the standard algorithm.

Key Points

It may be helpful to have students "build" (draw) the semi concrete models on graph paper, paying close

attention to the amount of boxes per "one" unit.

- ✓ In Example & We Do #1, both the dividend & divisor are whole numbers. The divisor is the number of groups we are sharing equally into. The quotient is the amount in 1 group. In this case, the amount in each group is going to be greater than one. However, if the divisor is bigger than the dividend, the amount in each group is going to be less than one whole.
- ✓ In Example & We Do #2, the dividend is a fractional number or decimal and the divisor is a whole number. The divisor is still the number of groups, and the quotient is the amount in 1 group. However, it's important to note that the amount in each group MUST be less than one whole, since the original dividend is less than one.
- ✓ In Example & We Do #3, both the dividend AND the divisor are fractional numbers or decimals. Therefore, we are *interested* in the amount in one fraction of the whole, but ultimately need to know how much in an ENTIRE group. This is a great example of why multiplying by the reciprocal works!

- ✓ While concrete & semi-concrete models are great for these types of problems for ***conceptual understanding***, this method is not recommended for numbers much smaller than those shown here.
- ❖ In Example #3, pay close attention to the size of the group. Specifically, in one third of the group, there are $\frac{4}{3}$, so in the entire circle, there are $\frac{12}{3}$, which is the entire group.
- ❖ In We Do #3, pay close attention to the size of the group. Specifically, in one third of the group, there are $\frac{7}{3}$, so in the entire circle, there are $\frac{21}{3}$, which is the entire group.

Warm-Up

CST: 6NS 2.2/CCSS 5.NF.3

What is the product of $\frac{2}{5}$ and $\frac{4}{5}$?

A $\frac{1}{5}$

B $\frac{8}{25}$

C $\frac{1}{2}$

D $\frac{6}{5}$

Review: Grade 6

If you and 3 friends earned $31 for helping a neighbor clean out his garage, how much money would each of you get, if you share the money equally?

Show 2 ways to find a solution.

Current: Grade 6/ 6.NS.3

28.35 ÷ 3

What are some common errors students make when solving this type of problem?

Other: Grade 4/ 4.OA.3

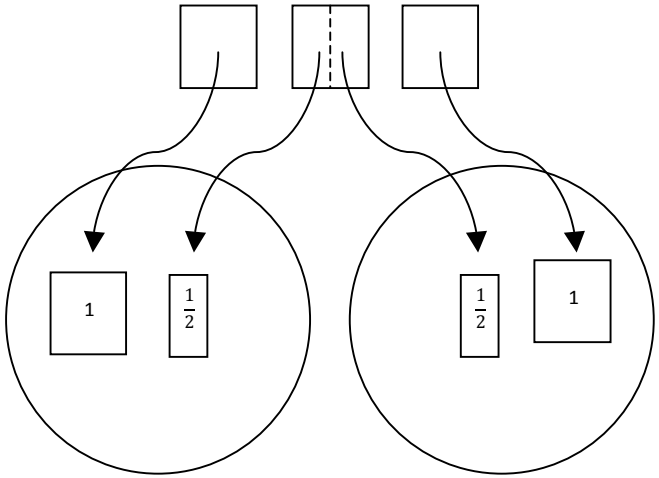
At a new school, there are 1,260 lockers to be installed in 14 hallways. How many lockers will be in each hallway, if each hallway has the same number of lockers?

In addition to solving this problem, come up with 3 possible distracters for a multiple choice question.

Example #1) $3 \div 2$

Share Equally

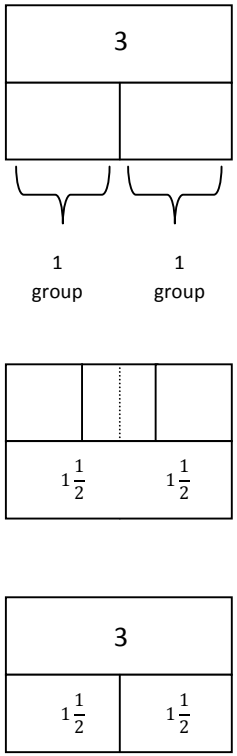
$3 \div 2$



$\therefore 3 \div 2 = 1\frac{1}{2}$ *or* 1.5

Bar model

$3 \div 2$



$\therefore 3 \div 2 = 1\frac{1}{2}$

2 groups of $1\frac{1}{2}$

Traditional

$3 \div 2$

$= \frac{3}{1} \div \frac{2}{1}$

$= \frac{3}{1} \cdot \frac{1}{2}$

$= \frac{3}{2}$

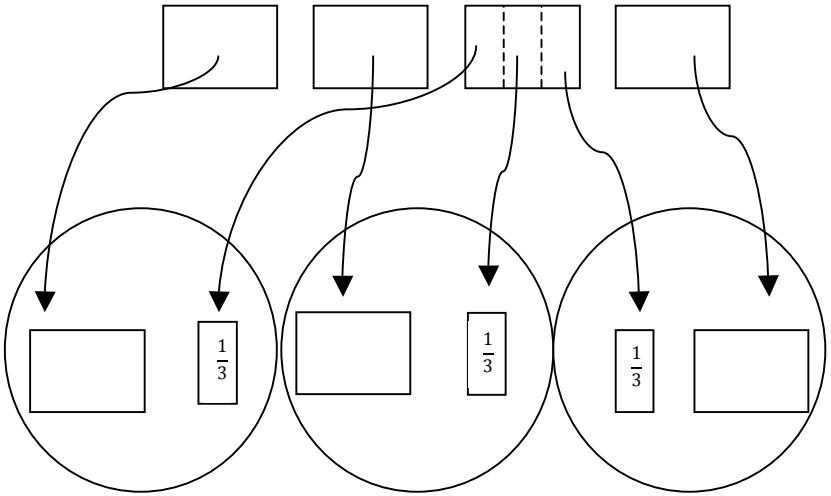
$= \frac{2}{2} + \frac{1}{2}$

$= 1\frac{1}{2}$

We Do #1) $4 \div 3$

Share Equally

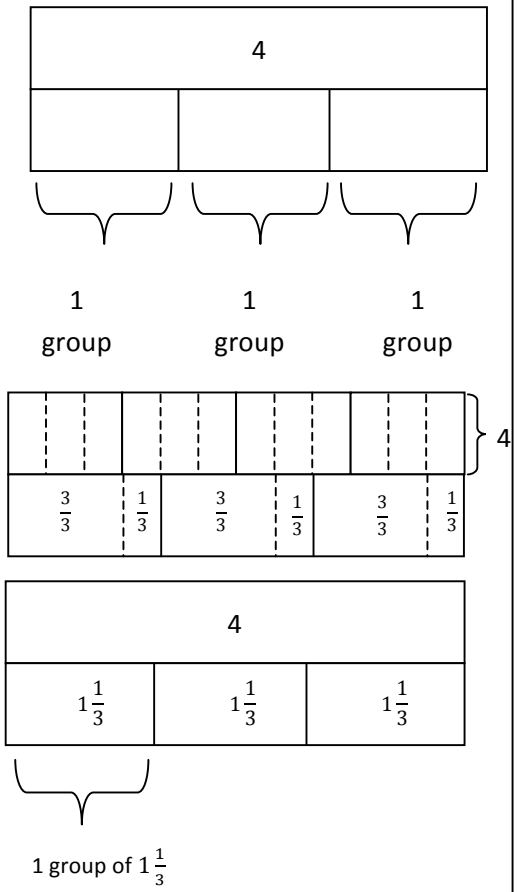
$4 \div 3$



$\therefore 4 \div 3 = 1\frac{1}{3}$ or $1.3\overline{3}$

Bar Model

$4 \div 3$



$\therefore 4 \div 3 = 1\frac{1}{3}$ or $1.3\overline{3}$

Traditional

$4 \div 3$

$= \frac{4}{1} \div \frac{3}{1}$

$= \frac{4}{1} \cdot \frac{1}{3}$

$= \frac{4}{3}$

$= \frac{3}{3} + \frac{1}{3}$

$= 1\frac{1}{3}$

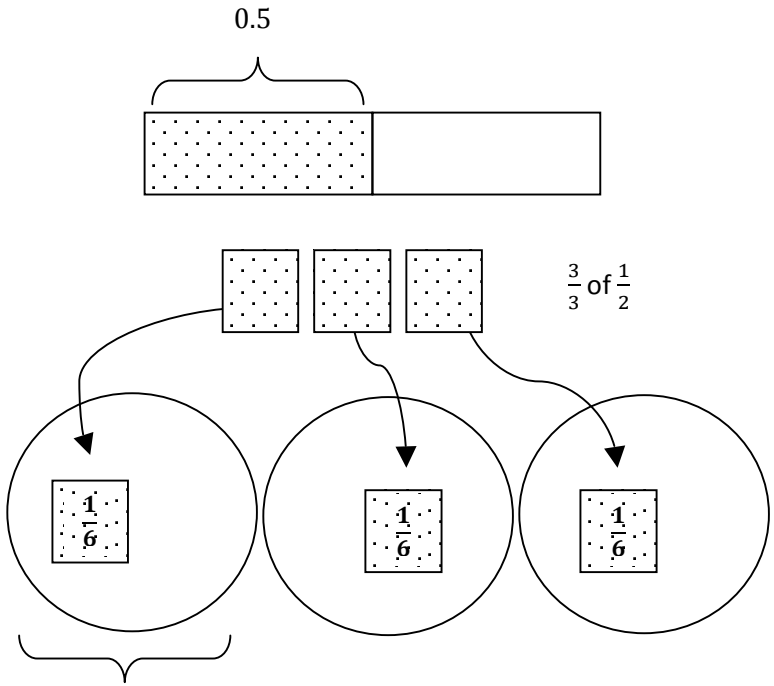
You Try #1) $7 \div 3$

Share Equally	Bar Model	Traditional
$7 \div 3$	$7 \div 3$	$7 \div 3$

Example #2) $0.5 \div 3$

Share Equally

$0.5 \div 3$

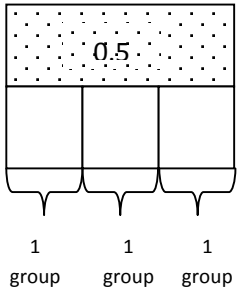


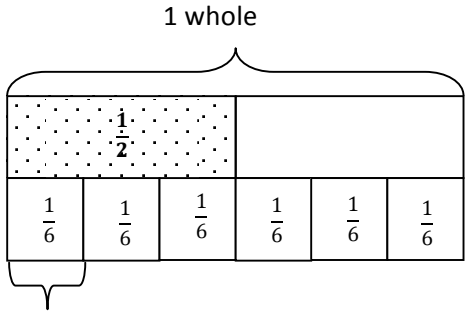
1 group of $\frac{1}{6}$

$\therefore 0.5 \div 3 = \frac{1}{6}$

Bar Model

$0.5 \div 3$





3 groups of $\frac{1}{6}$ or $0.16\overline{6}$ in 0.5

$\therefore 0.5 \div 3 = \frac{1}{6}\ or\ 0.16\overline{6}$

Traditional

$0.5 \div 3$

$= \frac{1}{2} \div 3$

$= \frac{1}{2} \div \frac{3}{1}$

$= \frac{1}{2} \cdot \frac{1}{3}$

$= \frac{1}{6}$

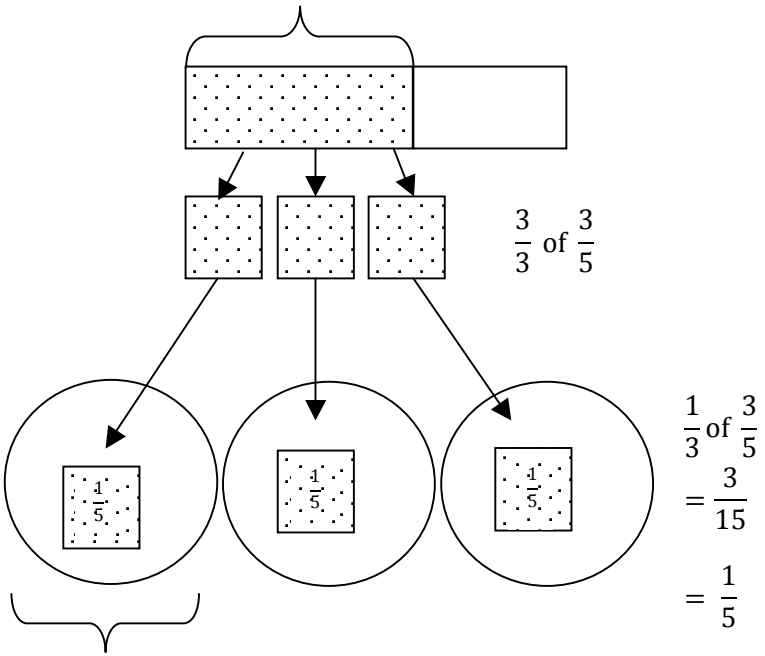
$\therefore 0.5 \div 3 = \frac{1}{6}$

We Do #2) $0.6 \div 3$

Share Equally

$0.6 \div 3$

$0.6 = \frac{3}{5}$



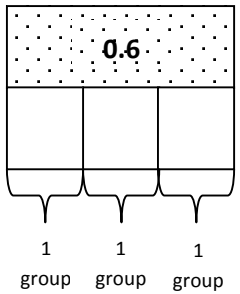
$\frac{1}{3}$ of $\frac{3}{5}$
$= \frac{3}{15}$
$= \frac{1}{5}$

1 group of $\frac{1}{5}$ or 0.2

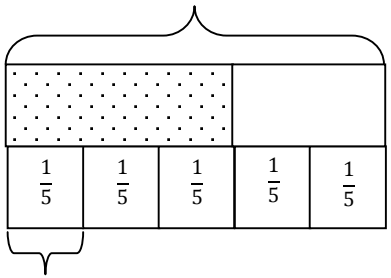
$\therefore 0.6 \div 3 = \frac{1}{5}$ *or* 0.2

Bar Model

$0.6 \div 3$



1 whole



3 groups of $\frac{1}{5}$ or 0.2 in 0.6

$\therefore 0.6 \div 3 = \frac{1}{5}$ *or* 0.2

Traditional

$0.6 \div 3$

$0.6 = \frac{3}{5}$

$= \frac{3}{5} \div 3$

$= \frac{3}{5} \div \frac{3}{1}$

$= \frac{3}{5} \cdot \frac{1}{3}$

$= \frac{3}{15}$

$= \frac{\cancel{3}^{1}}{\cancel{3}_{1} \cdot 5}$

$= \frac{1}{5}$

$\therefore 0.6 \div 3 = \frac{1}{5}$ *or* 0.2

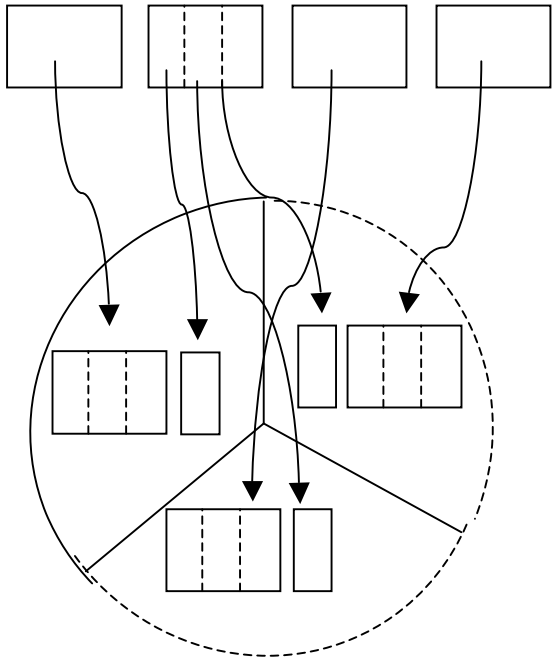
You Try #2) $0.5 \div 4$

Share Equally	Bar Model	Traditional
$0.5 \div 4$	$0.5 \div 4$	$0.5 \div 4$

Example # 3) $4 \div \frac{1}{3}$

Share Equally

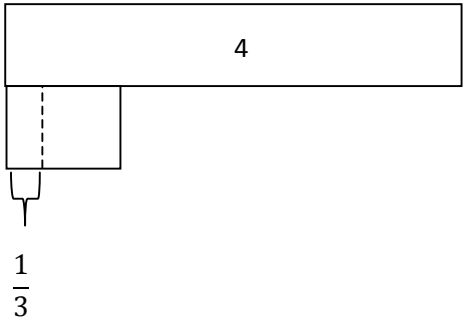
$$4 \div \frac{1}{3}$$



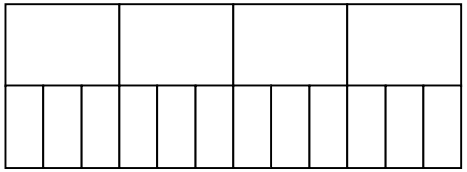
$$\therefore 4 \div \frac{1}{3} = 12$$

Bar Model

$$4 \div \frac{1}{3}$$



$\frac{1}{3}$



12 “one thirds” in 4

$$\therefore 4 \div \frac{1}{3} = 12$$

Traditional

$$4 \div \frac{1}{3}$$

$$= \frac{4}{1} \div \frac{1}{3}$$

$$= \frac{4}{1} \cdot \frac{3}{1}$$

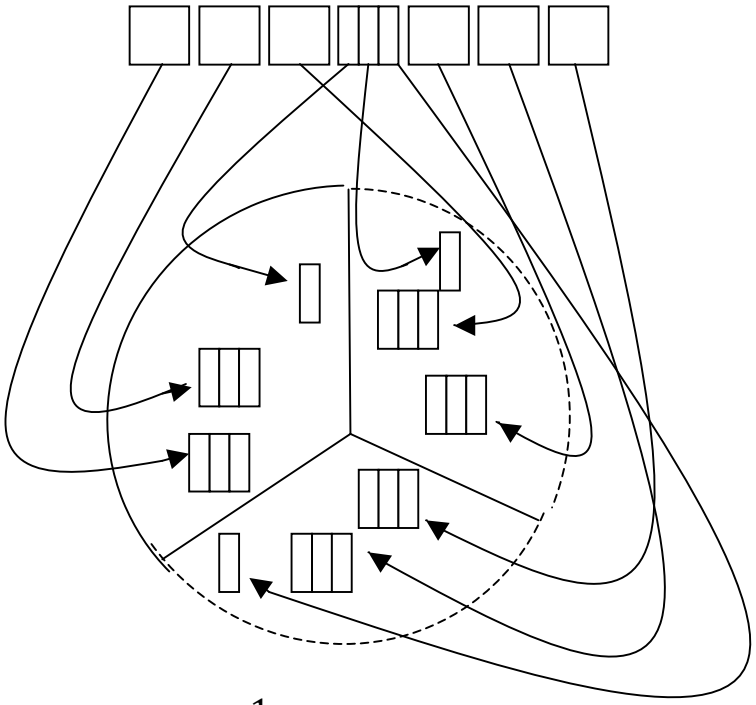
$$= \frac{12}{1}$$

$$= 12$$

We Do #3) $7 \div \frac{1}{3}$

Share Equally

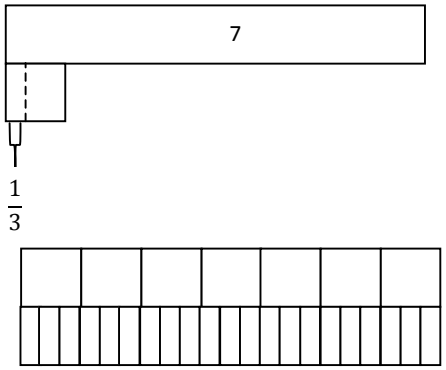
$$7 \div \frac{1}{3}$$



$$\therefore 7 \div \frac{1}{3} = 21$$

Bar Model

$$7 \div \frac{1}{3}$$



21 "one thirds" in 7

$$\therefore 7 \div \frac{1}{3} = 21$$

Traditional

$$7 \div \frac{1}{3}$$

$$= \frac{7}{1} \div \frac{1}{3}$$

$$= \frac{7}{1} \cdot \frac{3}{1}$$

$$= \frac{21}{1}$$

$$= 21$$

You Try #3) $4 \div 0.5$

Share Equally	Bar Model	Traditional
$4 \div 0.5$	$4 \div 0.5$	$4 \div 0.5$